5[65-02, 65F10, 65H10].—JOZSEF ABAFFY & EMILIO SPEDICATO, *ABS Projection Algorithms: Mathematical Techniques for Linear and Nonlinear Equations*, Ellis Horwood Series in Mathematics and its Applications, Wiley, New York, 1989, 220 pp., $24\frac{1}{2}$ cm. Price \$59.95.

The name ABS is composed of the initials of Abaffy, Broyden, and Spedicato, the three main contributors to the development of a unified theory for a rather large class of numerical algorithms for solving systems of m (linear and/or nonlinear) equations in n unknowns, where m is less than or equal to n. In the linear case these algorithms can be viewed as iterative procedures having the property that the vector produced at the ith iterate is a solution of the first i equations. Hence a solution of the original linear system is obtained after at most m iterations. The idea to solve the first i equations in i iterations is clearly not new. In the linear case this was used by the so-called escalator method a long time ago, and in the nonlinear case by Brown's algorithm which precedes by ten years the first paper of Abaffy from 1979. The escalator method was never recommended because of its higher computational complexity and numerical instability and, in fact, the above-mentioned paper of Abaffy was motivated by Huang's algorithm from 1975. There are other algorithms belonging to the ABS class that have been developed prior to Abaffy's paper. Among the most relevant we mention the method's of Pyle (1964) and Sloboda (1978) for linear systems, and the methods of Brent (1973), Cosnard (1975) and Gay (1975) for nonlinear systems.

The generality of the ABS class is due to a number of free parameters that can be chosen by different implementations in order to achieve different purposes. Typically, at step i of an ABS algorithm, one has a current approximation of the solution and a current value of some "projection matrix". The new approximation of the solution is obtained through line search along a certain search direction such that the ith equation is satisfied. The search direction itself can be arbitrarily chosen from the range of the transpose of the "projection matrix", as long as it is not perpendicular on the projection of the vector formed by the coefficients of the ith equation. Hence the first "free parameter". Finally, the "projection matrix" itself is updated via a rank-one update which involves a second "free parameter". The whole construction resembles the one employed for quasi-Newton methods in nonlinear optimization. A further generalization is obtained by considering at each iteration a new parameter vector, called the scaling vector. The scaled ABS algorithm is equivalent to the generalized conjugate direction algorithm for linear systems of Stewart (1973) in the sense that the two algorithms generate the same search directions and approximations to the solution, but they differ in their algebraic formulation, so that their numerical behavior may differ.

The book, overall, is very well written, the exposition is clear, the proofs are elegant, and a number of carefully worded bibliographical remarks clarify the relationship between the algorithms of the ABS class and related algorithms.

In the preface the authors express their hope that the monograph justifies their opinion that "not only the ABS approach is a theoretical tool unifying many algorithms which are scattered in the literature but also that it provides promising and, in some cases, proven effective techniques for computationally better algorithms for a number of problems". While being in complete agreement with the authors on the first statement, we have some reservation concerning the second one. This does not mean that no competitive software for linear and nonlinear systems could be based on the ABS approach. We only want to say that there is still a lot of work to be done.

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6[65H10, 65L10, 65M60, 58C27].—HANS D. MITTELMANN & DIRK ROOSE (Editors), *Continuation Techniques and Bifurcation Problems*, International Series of Numerical Mathematics, Vol. 92, Birkhäuser, Basel, 1990, 218 pp., 24 cm. Price $52.00.

This volume of invited articles addresses aspects of the computational analysis of parameter-dependent nonlinear equations. Twenty-seven authors, all active in the field, have contributed thirteen papers which may be loosely categorized into four groups.

The first group concerns numerical continuation techniques for problems with one real parameter. E. L. Allgower, C. S. Chie, and K. Georg discuss continuation in the case of large sparse systems. Continuation methods for variational inequalities are treated in papers by E. Miersemann and H. D. Mittelmann, and—using multigrid approaches—by R. H. W. Hoppe and H. D. Mittelmann. The applications of continuation procedures to partial differential equations modeling semiconductor devices are studied by W. M. Coughran, Jr., M. R. Pinto, and R. K. Smith. A continuation process involving damped Newton methods is analyzed by R. E. Bank and H. D. Mittelmann.

The papers in the second group involve symmetry in the study of bifurcation phenomena. The computation of symmetry-breaking bifurcation points for a class of semilinear elliptic problems is discussed by C. Budd. M. Dellnitz and B. Werner show how group-theoretic methods can be employed in the detection of bifurcation points and the computation of (multiple) Hopf points. For a two-parameter problem, symmetry is used by A. Spence, K. A. Cliffe, and A. D. Jepson in the computational determination of branches of Hopf points.

The third group consists of papers on the computational analysis of higher singularities. A. Griewank and G. W. Reddien develop a method for the computation of cusp catastrophes for steady-state operator equations and their discretizations. Numerical experiments on the interaction between fold points and